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STRENGTH CALCULATION FOR A BLADE AS AN ORTHOTROPIC PLATE OF LINEARLY VARIABLE THICKNESS

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A method of determining stresses in an orthotropic fan blade is proposed. Calculation results are compared with those obtained by the engineering method and with experimental data. It is shown that the stresses calculated with the use of the model proposed agree well with experimental results.

One of the calculated performance and reliability criteria for the main functional structural elements of machines is the static-strength safety factor [1]. The accuracy of its determination depends on the adequacy of the model and the method of strength calculation.

One of the main functional elements of fans used to ventilate metro tunnels [2] is a blade made from a composite material (DSV-4-R-2M press-material of quality "P"). This material is characterized by high strength, high vibration resistance, high resistance to attack by corrosive media, and high workability and it is relatively cheap.

Figure 1 shows the structure of the blade in three projections. For the stiffness and strength purposes, the reinforcing element 1 (filler) is shaped like a bundle of elementary fiberglass filaments ($d \leqslant 11$ μm) packed in different directions. Phenolformaldehyde resin is used as a binder 2 (matrix).

The advantages of these blades over sheet metal blades are a smaller labor input upon production, a smaller mass, higher corrosion resistance, and high aerodynamic performances.

Figure 2 shows the external contour of the fan blade 1 and its cross section 2 (ω is the angular velocity, φ is the angle of slope of the middle surface of the blade to the plane of rotation, $r_{\rm b}$ is the radius of the blade base, $2b_0$ is the width of the blade base, h is the height of the blade, and F_x and F_y are the centrifugal-force components).

Birger, Shorr, and Iosilevich [3] proposed a method of analyzing the turbomachinery parts, including turbine and compressor blades; the geometry of these blades and the fixing and loading conditions for them differ from those for fan blades.

In turbine blades of constant cross section, the maximum stress $\sigma_{\max}$ caused by centrifugal forces occurs in the root section [4]:

$$\sigma_{\max} = \frac{\rho\omega^2}{2\pi} S, \tag{1}$$

where ρ is the density the blade material, $S = \pi(R^2 - r_{\rm b}^2)$ is the area of the setting part of the impeller, and R is the outer radius of the blade rim.

For turbine blades with a curved surface and a variable cross-sectional area, the magnitude and distribution of tensile stresses depend on the law of variation of the cross-sectional area. If the cross-sectional area increases abruptly at the fixture, the maximum stresses occur near the middle cross section of the blade rather than in the root section [4].

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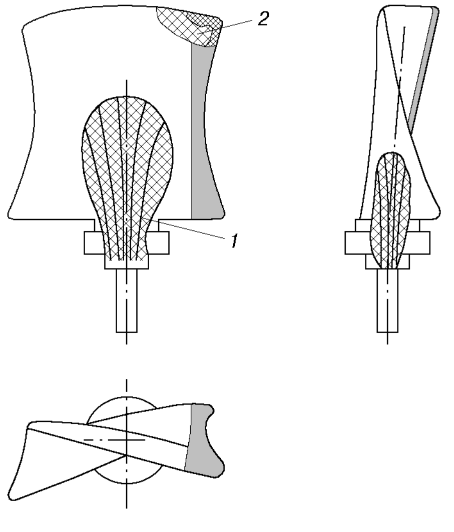

Fig. 1

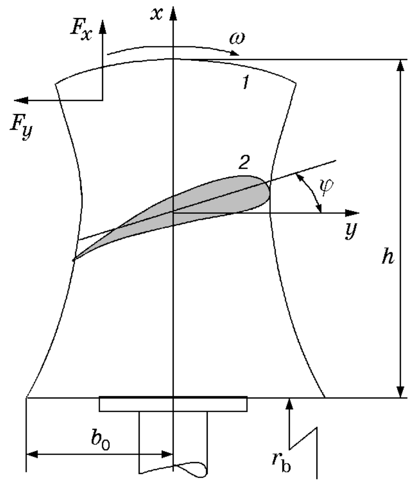

Fig. 2

The fiberglass blade of a VOVR-21 fan is characterized by a linearly variable cross-sectional area and a slightly curved helicoidal aerodynamic surface. Therefore, in determining the stresses caused by centrifugal forces, the blade is regarded as an orthotropic plate with a linearly variable thickness and a partly clamped contour. The centrifugal force is assumed to be distributed over the bulk of the blade. The curvature of the aerodynamic (external) surface is ignored.

The centrifugal-force components F_x and F_y (Fig. 2) corresponding to the x and y axes are given by $F_x = \rho\omega^2 x$ and $F_y = \rho\omega^2 y\cos\varphi$, respectively.

To calculate the stresses in the VOVR-21 blades subjected to centrifugal forces, we use:

— relations of Hook's generalized law ($\sigma_z = 0$) for an orthotropic plate

$$\varepsilon_x = \frac{\sigma_x}{E_x} - \frac{\nu_y}{E_y}\,\sigma_y, \qquad \varepsilon_y = \frac{\sigma_y}{E_y} - \frac{\nu_x}{E_x}\,\sigma_x, \qquad \gamma_{xy} = \frac{1}{G}\,\tau_{xy}; \tag{2}$$

— strain-continuity equation

$$\frac{\partial^2\varepsilon_x}{\partial y^2} + \frac{\partial^2\varepsilon_y}{\partial x^2} = \frac{\partial^2\gamma_{xy}}{\partial x\,\partial y}; \tag{3}$$

— equations of equilibrium

$$\frac{\partial\sigma_x}{\partial x} + \frac{\partial\tau_{xy}}{\partial y} + F_x = 0, \qquad \frac{\partial\tau_{xy}}{\partial x} + \frac{\partial\sigma_y}{\partial y} + F_y = 0. \tag{4}$$

In Eqs. (2)–(4), E_x and E_y are Young's moduli of the orthotropic material of the blade in the longitudinal and transverse directions, respectively, G is the shear modulus, and ν_x and ν_y are Poisson's ratios in the longitudinal and transverse directions, respectively.

With allowance for (2) and differentiation with respect to x and y, the strain-compatibility equation (3) can be written in the form

$$\frac{2}{E_x}\,\frac{\partial^2\sigma_x}{\partial y^2} + \Big(\frac{1}{G} - \frac{2\nu_y}{E_y}\Big)\frac{\partial^2\sigma_y}{\partial y^2} + \frac{2}{E_y}\,\frac{\partial^2\sigma_y}{\partial x^2} + \Big(\frac{1}{G} - \frac{2\nu_x}{E_x}\Big)\frac{\partial^2\sigma_x}{\partial x^2} + \frac{1}{G}\Big(\frac{\partial F_x}{\partial x} + \frac{\partial F_y}{\partial y}\Big) = 0. \tag{5}$$

For the fiberglass blade of the VOVR-21 as a plate of variable thickness, Eq. (4) for unit forces has the form

$$\frac{\partial s_x}{\partial x} + \frac{\partial t_{xy}}{\partial y} + tF_x = 0, \qquad \frac{\partial t_{xy}}{\partial x} + \frac{\partial s_y}{\partial y} + tF_y = 0, \tag{6}$$

where $s_x = t\sigma_x$, $s_y = t\sigma_y$, and $t_{xy} = t\tau_{xy}$ (t is the plate thickness). To satisfy Eqs. (6), we express the unit forces s_x, s_y, and t_{xy} in terms of the stress function $\Phi(x, y)$:

$$s_x = \frac{\partial^2 \Phi}{\partial y^2} - t\rho\omega^2 \frac{x^2}{2}, \qquad s_y = \frac{\partial^2 \Phi}{\partial x^2} - t\rho\omega^2 \frac{y^2}{2} \cos\varphi, \qquad t_{xy} = -\frac{\partial^2 \Phi}{\partial x\, \partial y}. \tag{7}$$

After appropriate manipulations, Eq. (5) becomes

$$\frac{1}{E_y}\frac{\partial^4 \Phi}{\partial x^4} + \Big(\frac{1}{G} - \frac{\nu_y}{E_y} - \frac{\nu_x}{E_x}\Big)\frac{\partial^4 \Phi}{\partial x^2 \partial y^2} + \frac{1}{E_x}\frac{\partial^4 \Phi}{\partial y^4} = \Big[\Big(\frac{\nu_y}{E_y} - \frac{1}{2G}\Big)(-\cos\varphi)$$

$$- \Big(\frac{\nu_x}{E_x} - \frac{1}{2G}\Big) - \frac{1}{2G}(1+\cos\varphi)\Big] t\rho\omega^2. \tag{8}$$

Using Eqs. (6), with allowance for (7) we express the stress components σ_x, σ_y, and τ_{xy} in the form

$$\sigma_x = \frac{s_x}{t} = \frac{1}{t}\frac{\partial^2 \Phi}{\partial y^2} - \rho\omega^2 \frac{x^2}{2}, \qquad \sigma_y = \frac{s_y}{t} = \frac{1}{t}\frac{\partial^2 \Phi}{\partial x^2} - \rho\omega^2 \frac{y^2}{2}\cos\varphi, \qquad \tau_{xy} = \frac{t_{xy}}{t} = -\frac{1}{t}\frac{\partial^2 \Phi}{\partial x\, \partial y}. \tag{9}$$

The problem of stress determination in a blade under the action of centrifugal forces reduces to the solution of Eq. (8) subject to the following boundary conditions (see Fig. 2):

— in the zone of the blade root ($x = r_{\rm b}$ and $y = 0$),

$$\sigma_x = p = \frac{1}{t}\frac{\partial^2 \Phi}{\partial y^2} - \rho\omega^2 \frac{x^2}{2};$$

— along the root ($x = r_{\rm b}$) and peripheral ($x = r_{\rm b} + h$) rims except for the point $(r_{\rm b}, 0)$,

$$\sigma_x = 0, \qquad \frac{1}{t}\frac{\partial^2 \Phi}{\partial y^2} - \rho\omega^2 \frac{x^2}{2} = 0;$$

— along the axis of the blade ($y = b_0$),

$$\sigma_y = 0, \qquad \frac{1}{t}\frac{\partial^2 \Phi}{\partial x^2} - \rho\omega^2 \frac{y^2}{2}\cos\varphi = 0;$$

— along the root ($x = r_{\rm b}$) and peripheral ($x = r_{\rm b} + h$) rims of the blade and along its axis ($y = b_0$),

$$\tau_{xy} = 0, \qquad -\frac{1}{t}\frac{\partial^2 \Phi}{\partial x\, \partial y} = 0.$$

Equation (8) subject to the above-mentioned boundary conditions was solved by the finite-difference method. Figure 3 shows the operator for a finite-difference grid with the step λ. Replacing the partial derivatives in Eq. (8) by finite-difference relations, we obtain a finite-difference analog of the differential equation at the point (x, y)

$$\frac{1}{E_y}\Phi_s - \Big(\frac{4}{E_y} + 2b\Big)\Phi_k + \Big[6\Big(\frac{1}{E_y} + \frac{1}{E_x}\Big) + 4b\Big]\Phi_i - \Big(\frac{4}{E_y} + 2b\Big)\Phi_l + \frac{1}{E_x}\Phi_v$$

$$- \Big(2b + \frac{4}{E_x}\Big)\Phi_m - \Big(2b + \frac{4}{E_x}\Big)\Phi_n + \frac{1}{E_x}\Phi_u + b\Phi_o + b\Phi_r + \frac{1}{E_y}\Phi_t + b\Phi_q + b\Phi_p = a(x_i, y_i). \tag{10}$$

We write the boundary conditions in the finite-difference form

$$\frac{1}{t\lambda^2}(\Phi_n - 2\Phi_i + \Phi_m) - \rho\omega^2 \frac{r^2}{2} = p, \qquad \frac{1}{t\lambda^2}(\Phi_n - 2\Phi_i + \Phi_m) - \rho\omega^2 \frac{x^2}{2} = 0,$$

$$\frac{1}{t\lambda^2}(\Phi_l - 2\Phi_i + \Phi_k) - \rho\omega^2 \frac{y^2}{2}\cos\varphi = 0, \qquad -\frac{1}{4t\lambda^2}(\Phi_p - \Phi_r + \Phi_o - \Phi_q) = 0. \tag{11}$$

Writing Eqs. (10) and (11) for all the internal and boundary points and taking into account the symmetry of the blade (plate), we obtain a system of linear algebraic equations for $\Phi(x_i, y_i)$. Once the values of $\Phi(x_i, y_i)$ are determined from the solution of this system, one can calculate the stresses σ_x, σ_y, and τ_{xy} at the nodal points from formula (9).

A stress calculation was performed for a blade made from a DSV-4-R-2M press-material of quality “P” with a volume fraction of the filler equal to 0.65. The mechanical characteristics of the material are as

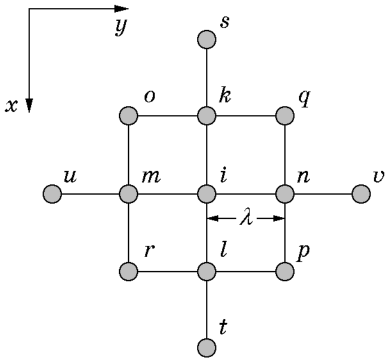

Fig. 3

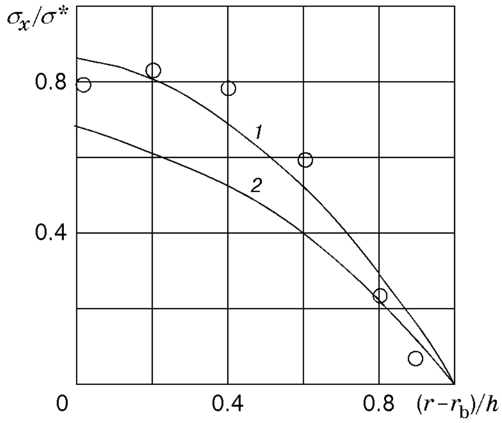

Fig. 4

follows: the density is $\rho = 1.7$ g/cm^3, the tensile strength is $\sigma_{\rm t} = 75$ MN/m^2, the compressive strength is $\sigma_{\rm c} = 13{,}000$ MN/m^2, Young's moduli in the longitudinal and transverse directions are $E_x = 52.1$ GN/m^2 and $E_y = 14$ GN/m^2, respectively, the shear modulus is $G = 6.3$ GN/m^2, and Poisson's ratios in the longitudinal and transverse directions are $\nu_x = 0.056$ and $\nu_y = 0.21$, respectively.

Figure 4 shows the stress distribution over the length of the blade. Curves 1 and 2 are plotted with the use of calculation results (curve 1 refers to our calculation and curve 2 to the that performed by the engineering method [1]). The stresses are normalized to the maximum stress in the root section of the blade $\sigma^* = r_c M\omega^2/A$, where M is the mass of the blade, A is the cross-sectional area in the zone of the blade root, and r_c is the distance from the axis of rotation of the impeller to the center of gravity of the blade. The points refer to the experimental data obtained by V. V. Vasil'ev at the Fedorov Institute of Rock Mechanics and Technical Cybernetics (Donetsk, Ukraine).

It follows from the results shown in Fig. 4 that the stresses calculated by the method proposed in this paper differ from the experimental values by no more than 11% on average. This error in determining the stresses is admissible in engineering calculation of fan blades. The maximum error in determining the stresses amounts to 19% and corresponds to the middle sections of the blade $(r - r_{\rm b})/h = 0.4$–0.5. The use of the engineering method [1] in determining the stresses in fiberglass blades leads to an average error of 26%.

To calculate the stresses in the cross sections of fiberglass blades with higher accuracy, one should take into account the nonlinear law of variation of the cross-sectional area of the blade and the structure of the composite material.

The model proposed allows one to choose rational variants of the blade design for a VOVR-21 impeller and similar blade machines.

REFERENCES

1. K. V. Frolov (ed.), *Design of Machines* [in Russian], Vol. 2, Mashinostroenie, Moscow (1994).
2. V. I. Solov'ev, Yu. G. Kuznetsov, and N. A. Popov, "Solution of the problems of ventilation of tunnels using an example of Novosibirsk metro," *Transp. Str.*, No. 9, 28–30 (1990).
3. I. A. Birger, B. F. Shorr, and G. B. Iosilevich, *Stress Analysis of Machine Elements* [in Russian], Mashinostroenie, Moscow (1993).
4. P. M. Varvaak, *Development and Application of the Grid Method to Calculation of Plates* [in Russian], Nauka, Moscow (1985).